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Radion Stabilization by Brane Matter

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Abstract

We find a static solution to Einstein's field equations on a five-dimensional orbifold with a compact S_1/Z_2 fifth direction and Poincare invariant $3+1$ sections. The solution describes a theory with bulk cosmological constant and 3-branes at the orbifold fixed points which carry matter density and pressure in addition to tension. The radius of the fifth dimension is determined by the matter content of the branes. The ratio of the space and time components of the metric depends on the fifth coordinate. Thus, the speed of propagation of massless fields is path dependent. For example, bulk and brane fields propagate with different speeds.

I. INTRODUCTION

A solution to the hierarchy problem has been proposed in which the observable universe is a 3-brane at an orbifold fixed point of the non-factorizable geometry

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2. \tag{1.1}$$

The orbifold has fixed points at $y = 0$ and $y = y_c$ where there are branes with tension $V_0 = 12k$ and $V_c = -12k$, respectively. We will refer to this as the RS model [1]. However, the dynamics does not determine the value of y_c, leaving it a free parameter. A solution to this so called "radion stabilization problem" has been found by adding a bulk scalar field, that is, one that has five-dimensional dynamics, to the model [2]. Alternative stabilization mechanisms have been proposed in [3].

In this paper we present a static solution of the field equations when the branes carry matter density and pressure in addition to tension. The solution fixes the radius of the

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fifth dimension. To see how this works let's first recall why there is a radion stabilization problem in the RS model. The warp factor in Eq. (1.1), $A = -k|y|$, has a constant slope and, therefore, the jump of the slope across the fixed point is $-2k$ regardless of where the jump occurs. Now if the warp factor had a non-constant slope $A'(y)$, then the jump $-2A'(y_c)$ does depend on y_c. Therefore the radius of the fifth dimension has to be chosen to accommodate the jump in energy density on the brane.

Adding matter to the basic setup of RS is necessary for a description of cosmology of the model. The cosmology of brane models has been investigated in a number of papers. A general formulation was given in Ref. [4]. The work in Refs. [5–7] is concerned with the cosmology of brane models of the Randall-Sundrum type [1]. These papers recognize that the jump in the warp factors across the branes as implied by Einstein's equations is enough to give the evolution of the scale factor in a FRW description of the cosmology on one brane. Therefore there has been little interest in the behavior of the metric in the bulk, see however Ref. [7]. It turns out that investigating bulk solutions can yield interesting results. The solutions we find exist only for a particular value of brane separation, which is determined by the matter content of the branes. Depending on the kind of matter such solutions can be either stable or unstable under small perturbations. The salient feature of our bulk solutions induced by brane matter, similar to the solutions in Ref. [8], is that the space and time components of the metric are not identical as they are in the RS model without matter. This means that the speed of propagation of massless fields depends on their trajectory. For example, bulk gravitons can propagate at a speed different than bulk photons. Finding out the strength of gravity and masses of bulk and brane fields in such complicated backgrounds is straightforward using the results of Ref. [9].

In Refs. [4–6] it is found that the scale factor a satisfies

$$\left(\frac{\dot a}{a}\right)^2 + \left(\frac{\ddot a}{a}\right) = \frac{1}{72}V(\rho - 3p) - \frac{1}{36}\rho(\rho + 3p), \tag{1.2}$$

where V, ρ and p stand for the tension, matter density and pressure on the brane respectively. We have set the five-dimensional gravitational constant to unity. Our static solution is not in contradiction with this brane equation because the solution demands that the three parameters V, ρ and p are such that the right hand side vanishes.

We present our solution in section II. We study non-static solutions as small perturbations to our solution in Sec. III and present our conclusions in section IV.

II. A STATIC SOLUTION WITH MATTER

We denote the coordinates of spacetime by x^A, $A = 0, \dots, 4$, and often use $t = x^0$ and $y = x^4$. The fixed points are at $y = 0$ and $y = y_c$. The class of spherically symmetric metrics we study is parameterized by three functions of t and y only [4]

$$ds^2 = G_{AB}dx^A dx^B = n^2(t,y)dt^2 - a^2(t,y)d\vec{x}^2 - b^2(t,y)dy^2. \tag{2.1}$$

Fixing $y = 0$ ($y = y_c$) we see that the metric gives a flat FRW cosmology on the brane with scale factor $R_0(t') = a(t(t'), 0)$ ($R_c(t') = a(t(t'), y_c)$) where $dt' = n(t, 0)dt$ ($dt' = n(t, y_c)dt$). We will denote by $g_{\mu\nu}$, with $\mu, \nu = 0, \dots, 3$, the induced metric on the brane.

The action is

$$S = \int d^5x \sqrt{G}\,[-R-\Lambda] + \int d^4x \sqrt{-g}\,[-V_0]_{y=0} + \int d^4x \sqrt{-g}\,[-V_c]_{y=y_c}\,. \tag{2.2}$$

The constants Λ, V_0 and V_c represent the cosmological constant in the bulk (5-dimensional space) and on the branes at $y=0$ and $y=y_c$, respectively. In addition there is matter density and pressure on the branes, introduced into the field equations directly by their contribution to the energy momentum tensor:

$$T^{AB} = \tilde{T}^{AB} + \frac{S_0^{AB}}{b}\delta(y) + \frac{S_c^{AB}}{b}\delta(y-y_c), \tag{2.3}$$

where $\tilde{T}^{AB}$ is derived as usual by varying the action with respect to the metric, and S^{AB} are contributions from perfect fluids of density ρ_0 and ρ_c and pressures p_0 and p_c on the branes,

$$S_B^A = \mathrm{diag}\,(\rho, -p, -p, -p, 0). \tag{2.4}$$

Einstein's equations are

$$R^{AB} - \frac{1}{2}G^{AB}R = \kappa T^{AB}. \tag{2.5}$$

Here R^{AB} and R are the Ricci tensor and scalar. The gravitational constant is κ and we work in units of $\kappa = 1$.

For the particular metric (2.1) Einstein's equations are

$$3\left[\left((\frac{\dot{a}}{a})^2 + \frac{\dot{a}\,\dot{b}}{a\,b}\right) + \frac{n^2}{b^2}\left(-\frac{a''}{a} - (\frac{a'}{a})^2 + \frac{a'\,b'}{a\,b}\right)\right] = \frac{1}{2}n^2\Lambda + \delta(y)\frac{n^2}{b}(\frac{1}{2}V_0 + \rho_0) + \delta(y-y_c)\frac{n^2}{b}(\frac{1}{2}V_c + \rho_c) \tag{2.6}$$

$$3\left(\frac{\dot{a}\,n'}{a\,n} + \frac{a'\,\dot{b}}{a\,b} - \frac{\dot{a}'}{a}\right) = 0, \tag{2.7}$$

$$\frac{a^2}{n^2}\left(-\frac{\dot{a}^2}{a^2} - 2\frac{\ddot{a}}{a} + 2\frac{\dot{a}\,\dot{n}}{a\,n} - 2\frac{\dot{a}\,\dot{b}}{a\,b} - \frac{\ddot{b}}{b} + \frac{\dot{n}\,\dot{b}}{n\,b}\right) + \frac{a^2}{b^2}\left(\frac{a'^2}{a^2} + 2\frac{a''}{a} + 2\frac{a'\,n'}{a\,n} - 2\frac{a'\,b'}{a\,b} + \frac{n''}{n} - \frac{n'\,b'}{n\,b}\right)$$
$$= -\frac{a^2}{2}\Lambda - \delta(y)\,\frac{a^2}{b}(\frac{1}{2}V_0 - p_0) - \delta(y-y_c)\,\frac{a^2}{b}(\frac{1}{2}V_c - p_c), \tag{2.8}$$

$$3\left[\frac{b^2}{n^2}\left(-\frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a} + \frac{\dot{a}\,\dot{n}}{a\,n}\right) + \left(\left(\frac{a'}{a}\right)^2 + \frac{a'\,n'}{a\,n}\right)\right] = -\frac{1}{2}b^2\Lambda. \tag{2.9}$$

Here a dot is a shorthand for $\partial/\partial t$ and a prime for $\partial/\partial y$. The first four equations correspond to the 00, 04, 11 and 44 components of Einstein's equations. Conservation of the stress-energy tensor gives

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \tag{2.10}$$

on each brane.

We look for static solutions to these equations. One may then reparametrize the fifth coordinate to enforce $b = 1$. The resulting equations are most easily solved in terms of the warp factors A and N defined by

$$a = \exp(A) \qquad n = \exp(N). \tag{2.11}$$

For convenience we also introduce $k \equiv \sqrt{-\Lambda/12}$. In the bulk Einstein equations reduce to

$$A'' + 2A'^2 = 2k^2, \tag{2.12}$$
$$2A'' + 3A'^2 + 2A'N' + N'' + N'^2 = 6k^2, \tag{2.13}$$
$$A'^2 + A'N' = 2k^2. \tag{2.14}$$

The solution is straightforward. The metric in the bulk is given, for $y \geq 0$, by

$$a^2 = a_*^2 \cosh[2k(y - y_*)], \tag{2.15}$$
$$n^2 = n_*^2 \sinh[2k(y - y_*)] \tanh[2k(y - y_*)], \tag{2.16}$$
$$b^2 = 1, \tag{2.17}$$

where y_*, n_* and a_* are constants. For related bulk solutions see Ref. [10].

To complete the solution we must examine the field equations at the brane. The delta functions on the right hand side of Eqs. (2.6) and (2.8) must be saturated on the right hand side by the second derivatives. To this effect we demand

$$2A'|_{y=0+} = -\frac{1}{6}V_0 - \frac{1}{3}\rho_0, \tag{2.18}$$
$$-2A'|_{y=y_c-} = -\frac{1}{6}V_c - \frac{1}{3}\rho_c, \tag{2.19}$$
$$2(2A' + N')|_{y=0+} = -\frac{1}{2}V_0 + p_0, \tag{2.20}$$
$$-2(2A' + N')|_{y=y_c-} = -\frac{1}{2}V_c + p_c. \tag{2.21}$$

The solution to these are two equations of fine tuning,

$$144k^2 = (V_0 + 2\rho_0)(V_0 - \rho_0 - 3p_0) = (V_c + 2\rho_c)(V_c - \rho_c - 3p_c), \tag{2.22}$$

and two equations fixing the parameters y_* and y_c:

$$12k \tanh[2ky_*] = V_0 + 2\rho_0, \qquad 12k \tanh[2k(y_c - y_*)] = V_c + 2\rho_c. \tag{2.23}$$

The first two conditions, Eq. (2.22) are similar in spirit to the two fine tuning equations in the RS model that set the square of the tension in each brane in a fixed proportion to the bulk cosmological constant. By comparison, the stabilization mechanism of Refs. [2,11] requires only one fine tuning, which is equivalent to setting the cosmological constant to zero. The last two equations describe what was alluded to in the introduction, that in order for the warp factor to jump by the appropriate amount the location of the brane must be chosen accordingly. Hence the radius is stabilized, or, should we say, equilibrated (stability is investigated in the next section).

Note that positive tension branes ($V > 0$) can only exist for $p \leq -\rho$. If $\frac{1}{3}\rho > p > -\frac{1}{3}\rho$ on both branes, solutions exist only for $V_0 < -12k$ and $V_c < -12k$. But if $p > \frac{1}{3}\rho$, V can be larger or smaller than $-12k$.

If we insist, as in the RS model, that $V_0^2 = V_c^2 = 144k^2$ then the conditions (2.22) are identical to the static solution conditions in Eq. (1.2). We recover the RS model taking vanishing matter density and pressure and the limit $y_* \to \infty$.

However we need not insist on imposing $V_0^2 = V_c^2 = 144k^2$. In fact, we are now free to chose any tension provided the conditions (2.22) are satisfied and a solution to Eqs. (2.23) with $y_c > 0$ can be found.

III. SMALL PERTURBATIONS

Armed with the new solutions with static matter density, we proceed to investigate the time dependence of small matter perturbations. Let us denote the static solution of the previous section by $n_0 = e_0^N$, $a_0 = e_0^A$, and $b_0 = 1$. We look for solutions to the field equations, Eqs. (2.6)–(2.9), of the form

$$\begin{aligned} n &= n_0(1+\delta n), \\ a &= a_0(1+\delta a), \\ b &= b_0(1+\delta b). \end{aligned} \tag{3.1}$$

In addition we set the density on the branes to $\rho_0 + \delta\rho_0$ and $\rho_c + \delta\rho_c$ and the pressure to $p_0 + \delta p_0$ and $p_c + \delta p_c$.

We count orders of the perturbative expansion parametrically in $\delta\rho$ and δp. That is, we re-scale $\delta\rho \to \epsilon\delta\rho$, count powers of ϵ and set $\epsilon = 1$ at the end of the calculation. In particular this implies that we make no assumption as to the relative importance of temporal or spatial derivatives [6].

To derive the linearized equations in the bulk, we use the parameterization in Eqs. (3.1). The 00, 04, 11 and 44 components of Einstein's equations give

$$\delta a'' + A_0'(4\delta a' - \delta b') = 4k^2\delta b, \tag{3.2}$$

$$\frac{\partial}{\partial t}\left((N_0' - A_0')\delta a + A_0'\delta b - \delta a'\right) = 0, \tag{3.3}$$

$$\begin{aligned} 2\delta a'' + \delta n'' + (6A_0' + 2N_0')\delta a' + 2(A_0' + N_0')\delta n' \\ -(2A_0' + N_0')\delta b' - \frac{1}{n_0^2}\left(2\delta\ddot{a} + \delta\ddot{b}\right) = 12k^2\delta b, \end{aligned} \tag{3.4}$$

$$(2A_0' + N_0')\delta a' + A_0'\delta n' - \frac{1}{n_0^2}\delta\ddot{a} = 4k^2\delta b. \tag{3.5}$$

The solution to these equations gives δb and $\delta n'$ in terms of δa:

$$\delta b = \frac{1}{A_0'}\left[F + \delta a' + (A_0' - N_0')\delta a\right], \tag{3.6}$$

$$\delta n' = \frac{1}{A_0'}\left[\frac{1}{n_0^2}\delta\ddot{a} - (2A_0' + N_0')\delta a' + 4k^2\delta b\right]. \tag{3.7}$$

In Eq. (3.7) δb is understood as shorthand for the solution of Eq. (3.6). In Eq. (3.6) F is a function of y satisfying

$$F' - \frac{A_0'' - 4k^2}{A_0'} F = 0. \tag{3.8}$$

It must be observed that F may be discontinuous at $y = 0$. In fact, continuity of δb at $y = 0$ requires $F(0+) + F(0-) = 0$. The solution to Eq. (3.8) is $F(y) = F_* / \sinh(4k(y - y_*))$, with F_* a constant.

We connect the bulk solutions for $y > 0$ and $y < 0$ demanding continuity of the fields at the brane, $y = 0$, and using the jump equations for the discontinuous derivatives at $y = 0$. The latter give jump conditions for the perturbations

$$\delta a'|_{0+} = -\frac{1}{6}\left[(\frac{1}{2}V_0 + \rho_0)\delta b + \delta\rho_0\right], \tag{3.9}$$

$$(2\delta a' + \delta n')|_{0+} = \frac{1}{2}\left[(-\frac{1}{2}V_0 + p_0)\delta b + \delta p_0\right]. \tag{3.10}$$

Similarly, the jump equations at the second brane are

$$-\delta a'|_{y_c-} = -\frac{1}{6}\left[(\frac{1}{2}V_c + \rho_c)\delta b + \delta\rho_c\right], \tag{3.11}$$

$$-(2\delta a' + \delta n')|_{y_c-} = \frac{1}{2}\left[(-\frac{1}{2}V_c + p_c)\delta b + \delta p_c\right]. \tag{3.12}$$

In addition, conservation of energy gives, on the brane,

$$\delta\rho_0 + 3(\rho_0 + p_0)\delta a|_{y=0} = 0 \qquad \text{and} \qquad \delta\rho_c + 3(\rho_c + p_c)\delta a|_{y=y_c} = 0. \tag{3.13}$$

The right hand side of these equations could be a non-zero constant. However, we set it to zero since we are not interested in constant shifts in the mass density (since these are accounted for in the exact solution of Sec. II).

The jump equations at $y = 0$, Eqs. (3.9)–(3.10), determine $\delta a'$ in terms of δa and give an equation for δa, namely

$$\frac{1}{n_0^2 N_0'}\delta\ddot{a} + \frac{1}{2}\frac{A_0'}{N_0'}\delta p_0 - \frac{1}{6}\delta\rho_0 = 0 \tag{3.14}$$

The time dependence of the matter is fixed by the continuity equation, Eq. (3.13). Given an equation of state for the perturbations, $\delta p_0/\delta\rho_0 = w_0$, one can solve this equation. Let's rewrite the equation as

$$\delta\ddot{a} - \Gamma_0^2 \delta a = 0. \tag{3.15}$$

Then the coefficient Γ_0^2 can be expressed in terms of the initial parameters. In terms of $t_0 \equiv (V_0 + 2\rho_0)/12k = \tanh(2ky_*)$ we find

$$\Gamma_0^2 = 2k^2 n^2(y=0)\left[\frac{2}{t_0} - (3\omega_0 + 1)t_0\right]\left[\frac{1}{t_0} - t_0\right] \tag{3.16}$$

Hence, $\Gamma_0^2 > 0$ for equations of state with $w_0 < 1/3$. But for $w_0 > 1/3$ Γ_0^2 can be either positive or negative depending on the value of t_0 with $\omega_0 = \frac{1}{3}(\frac{2}{t_0^2} - 1)$ being the dividing line.

We see that the time dependence of δa on the $y = 0$ brane is exponential for $\omega_0 < 1/3$, but can be oscillatory for $\omega_0 > 1/3$:

$$\delta a|_{y=0} = c_0 e^{\Gamma_0 t} + d_0 e^{-\Gamma_0 t}, \tag{3.17}$$

where Γ_0 is either real or purely imaginary. Substituting this solution in the equation for $\delta a'$ gives

$$\delta a'|_{y=0} = 0. \tag{3.18}$$

This implies that the jump in F vanishes. Therefore $F(y) = 0$.

An entirely analogous solution is obtained on the brane at $y = y_c$ by replacing the subscript "0" in Eqs. (3.15)–(3.17) by the second brane subscript "c."

To extend the solution on the branes into the bulk we must fix the gauge. There is a gauge freedom, that is, reparametrization invariance consistent with the form of our metric. Starting from the metric

$$ds^2 = n^2(t', y')dt'^2 - a^2(t', y')d\vec{x}^2 - b^2(t', y')dy'^2, \tag{3.19}$$

we look for infinitesimal transformations

$$t' = t + T(t, y) \tag{3.20}$$
$$y' = y + Y(t, y) \tag{3.21}$$

that leave the form of the metric invariant and has fixed points at at $y = 0$ and $y = y_c$. Here T and Y are infinitesimal. The only constraints on these functions come from the absence of off-diagonal terms in the metric,

$$n^2 T' - b^2 \dot{Y} = 0, \tag{3.22}$$

and from the fixed points,

$$Y(t, 0) = Y(t, y_c) = 0. \tag{3.23}$$

Under the gauge transformation the metric variations are

$$\Delta\delta n = N_0' Y + \dot{T}, \tag{3.24}$$
$$\Delta\delta a = A_0' Y, \tag{3.25}$$
$$\Delta\delta b = Y'. \tag{3.26}$$

For simplicity we have indicated the variation about a static solution with $b_0 = 1$ and $\dot{a}_0 = \dot{n}_0 = 0$. We would like to use this gauge freedom to impose the gauge condition

$$\delta b(y, t) = 0. \tag{3.27}$$

However, this cannot be done in general since there is only one constant of integration in the gauge condition (3.26). Instead consider covering the space with two different patches. We label the metric perturbations δa_1, δn_1 and δb_1 on the patch defined by $0 \leq y < y_c/2 + \epsilon$ and by δa_2, δn_2 and δb_2 on the patch defined by $y_c/2 - \epsilon < y \leq y_c$. Given a solution with metric δa, δn and δb we choose the new coordinates by choosing

$$Y_1 = -\int_0^y \delta b(\hat{y}, t) d\hat{y} \qquad \text{and} \qquad Y_2 = -\int_{y_c}^y \delta b(\hat{y}, t) d\hat{y}. \tag{3.28}$$

Thus $\delta b_1 = 0$ for $0 \leq y \leq y_c/2 + \epsilon$ and $\delta b_2 = 0$ for $y \geq y_c/2 - \epsilon$.

The solution for δa_1 and δa_2 follows immediately from Eq. (3.6) setting $\delta b = 0$:

$$\delta a_1(y,t) = F_{*1}/4k + A_0' \xi_1(t) \qquad \text{and} \qquad \delta a_2(y,t) = F_{*2}/4k + A_0' \xi_2(t), \tag{3.29}$$

where $\xi_{1,2}$ are arbitrary functions of t but independent of y. We can now use our jump conditions to specify these completely:

$$\delta a_1(y,t) = \frac{A_0'(y)}{A_0'(0+)} \delta a|_{y=0+} \qquad \text{and} \qquad \delta a_2(y,t) = \frac{A_0'(y)}{A_0'(y_c-)} \delta a|_{y=y_c-}. \tag{3.30}$$

In the overlap region, $|y - y_c/2| < \epsilon$, these solutions are related by a gauge transformation:

$$\delta a_2 - \delta a_1 = A_0' \int_0^{y_c} \delta b(\hat{y}, t) d\hat{y}. \tag{3.31}$$

Thus we obtain

$$\begin{aligned} \int_0^{y_c} \delta b(\hat{y}, t) d\hat{y} &= \frac{\delta a}{A_0'}\Big|_{y=y_c-} - \frac{\delta a}{A_0'}\Big|_{y=0+} \\ &= \frac{c_c e^{\Gamma_c t} + d_c e^{-\Gamma_c t}}{k \tanh(2k(y_c - y_*))} + \frac{c_0 e^{\Gamma_0 t} + d_0 e^{-\Gamma_0 t}}{k \tanh(2k y_*)}, \end{aligned} \tag{3.32}$$

where it is understood that $\Gamma_{0,c}$ can be purely imaginary if $\omega_{0,c} > \frac{1}{3}$.

IV. DISCUSSION AND CONCLUSIONS

In Sec. II we gave a class of solutions to the field equations with matter on the two branes. The solutions are static. The price to pay for time independence is a fine tuning of the amount of energy on each brane, as expressed in Eq. (2.22). These fine tunings are no worse than the corresponding fine tunings in the RS model.

The salient feature of the solutions found in Sec. II is that the physical size of the fifth dimension is fixed. This suggested the exciting possibility that the radion stabilization problem is not an issue in models with matter on the branes.

However, static solutions are not cosmologically acceptable. Nevertheless, one wonders if even for non-static solutions with matter the radius is stabilized. Short of finding an exact non-static solution we have exhibited in Sec. III an approximate solution by linearizing the field equations around the static solutions.

The result of the linearized analysis is summarized by Eqs. (3.17) and (3.32). The first one establishes that the metric perturbations have exponential time dependence for equations of state with $\omega = p/\rho < 1/3$, but can be oscillatory for $\omega > 1/3$. The second gives the time dependence of the radius of the space. Indeed, a measure of the radius is

$$L = \int \sqrt{-G_{MN} dx^M dx^N} \tag{4.1}$$

along a line of constant t and $\vec{x}$:

$$L = y_c + \int_0^{y_c} \delta b(\hat{y}, t) d\hat{y}. \tag{4.2}$$

For $\omega < 1/3$ the radius grows exponentially, at least while the exponential is small enough that the linearized solution remains a good approximation. On the other hand, for $\omega > 1/3$ the radius can oscillate about the equilibrium value y_c or can grow exponentially depending on the value of $t_0 = \frac{V_0 + 2\rho_0}{12k}$.

One gains some understanding of the behavior of the scale factor a on the brane by considering the full, non-perturbative description of its evolution at an orbifold fixed point. Since we do not wish to insist that $V^2 = 144k^2$ we modify Eq. (1.2) to allow for an unconstrained tension,

$$\left(\frac{\dot{a}^2}{a^2}\right) + \left(\frac{\ddot{a}}{a}\right) = \frac{1}{72} V(\rho - 3p) - \frac{1}{36}\rho(\rho + 3p) + \frac{1}{72} V^2 - 2k^2, \tag{4.3}$$

Let the equation of state be $p = w\rho$. Using the equation of conservation, Eq. (2.10), one has $\rho = \rho_0 a_0^{3(1+w)} / a^{3(1+w)}$, where ρ_0 and a_0 are the density and scale factor at a fixed time. Then one can rewrite the equation for the form factor as the equation for a particle with displacement $r = a^2$ in a potential, $\ddot{r} = -U'(r)$, with

$$U(r) = C_1 r^{2 - \frac{3}{2}(1+w)} + C_2 r^{2-3(1+w)} + C_3 r^2, \tag{4.4}$$

where

$$C_1 = -\frac{\rho_0 a_0^{3(1+w)} V}{18} \tag{4.5}$$

$$C_2 = -\frac{\rho_0^2 a_0^{6(1+w)}}{18} \tag{4.6}$$

$$C_3 = 2k^2 - \frac{V^2}{72} \tag{4.7}$$

Recall that for $\omega > -1$ our solutions must have negative tension branes $V < 0$, so $C_1 > 0$ and $C_2 < 0$. For $\frac{1}{3} > \omega > -\frac{1}{3}$, $V < -12k$ and, therefore, C_3 is negative and the only extremum of $U(r)$ is a maximum. But for $w > \frac{1}{3}$, V can be larger or smaller than $-12k$ and, therefore, the sign of C_3 can be either positive or negative. The extremum of $U(r)$ can, therefore, be a maximum or minimum, in accordance with our analysis of small perturbations.

The physical interpretation of the solutions discussed here is not straightforward. One cannot, as in the case of the RS model, simply renormalize fields on either brane to cast their kinetic energy term in standard form, since $n \neq a$. Moreover, a null trajectory parallel to the branes has $dx/dt = n/a$. Since $n/a = (n_*/a_*)\tanh[2k(y-y_*)]$ differs between the two fixed points, one can arrange for superluminal signal travel. For example, on one brane one can send a graviton across to the other brane, relay the signal along the brane by photons, and then relay the signal back to the first brane via gravitons. Neglecting the time of travel between branes, we see that the interval between emission and reception can be shorter than the time for a photon to travel between the same two points directly on the first brane [12]. A complete discussion is the subject of a forthcoming paper.

Since for standard fluids in equilibrium the equation of state must have $\omega < 1/3$, the exponential growth of the scale factor and radius make this model an unlikely candidate for cosmology. However, it may be possible to stabilize the radius by modifying the model, say, by adding matter in the bulk, *e.g.*, scalar fields [2]. The cosmology of such a model may indeed be acceptable [6].

Note Added While this article was being completed three related works have been submitted to the archives. Ref. [13] discusses general solutions with different space and time components of the 5-d metric. We have not attempted to check if an explicit coordinate transformation can convert our bulk solution to a form used in Ref. [13]. The authors also discuss Lorentz symmetry violations due to different propagation speeds of bulk and brane fields. Ref. [14] uses supersymmetric gauge dynamics for stabilizing the radius and Ref. [15] uses the Casmir force due to a bulk scalar field.

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